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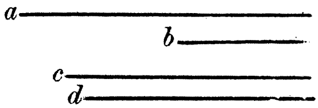
one edge of the pyramid, and on the sections of the other as upper bases prisms are constructed similarly.

"The sum of the first series of prisms is greater than $S-ABC$, and the sum of the second series is less than $S'-A'B'C'$; therefore, the difference between $S-ABC$ and $S'-A'B'C'$ is less than the difference between the sums of these two series of prisms."

The last conclusion quoted contains an unwarranted assumption, based on the supposition that if $a>b$ and $c>d$, the difference between a and d is greater than the difference between b and c which is not necessarily true. An illustration by means of lines also will show the fallacy. The error probably arose from overlooking the fact that so far as we know from the conditions assumed, there is no reason to suppose the second pyramid not greater than the first.



This proof, which closes by showing that the difference of the two series of prisms, being equal to the lowest prism of the first series, can be made less than any assignable volume by indefinitely increasing the number of prisms, and that the difference of the pyramids thus cannot be any assignable volume, replaces in an older edition a proof based on the theory of limits. Unless it was to relieve the monotony of proof by limits, it is not easy to see why the change was made.

I have also found the same proof in a recent geometry by another author.[‡] In a third[§] I find a similar idea, but used in a legitimate way. Here both sets of prisms are constructed on one pyramid and the limit of the sum of the inscribed series is thus shown to be the pyramid. Then by inscribing a series in each of the pyramids with equivalent bases and equal altitude, the method of limits is used to prove the pyramids equal. The other method may be considered an abbreviation of this, but its introduction of the objectionable feature ought to rule it out.

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REMARK ON DIVISION OF CONCRETE NUMBER.

By C. H. JUDSON, LL. D., Professor of Mathematics, Furman University, Greenville, South Carolina.

Professor Ellwood's article, in the February Number of the MONTHLY, suggests the question—What is a *Concrete Number?*

Number answers to the question—How many ? Quantity (quantus) answers to the question—How much ?

Twenty gallons is a concrete *expression*, representing *quantity*. It is complex, and consists of one expression which represents—how many, and another which represents a concrete unit of *quantity*. It is, therefore, something more than Number; unless we accept the Pythagorean notion—that "*Number* is the essence of *things*."

[‡] Stewart, Book IX., Prop. IX.

[§] Byerly's Chauvenet, Book VII. Props. XV. and XVI.

Most definitions and illustrations of *Concrete Numbers* are ambiguous. Prof. Simon Newcomb (see Alg. p. 3) says, "A concrete number is one in which the kind of *quantity* which it measures" (?) "is expressed or understood; as, Seven *miles*, Three *days*." (*Italics his.*)

Webster says, "Concrete number is a number *associated with*, or *applied to* a particular object; as, *Three* men, *five* days." Which of the two is correct? Do the italicised words of the former or of the latter express *numbers?* If the latter, then the so-called *Denominate Numbers* of our Arithmetics should be "Denominate Quantities."

Now, if we define division as a process of finding one of the equal parts of a *number*, may we not substitute *concrete quantity* for *number?* A fourth part of $20 is $5 and four times $5 are $20.

We wish to know what is a fourth part of 5768 acres. How shall we find it? What name shall we give to the process? Prof. Ellwood would say "Take a fourth part of the *number* 5768 and annex the name of the concrete unit.

We suggest that a *concrete number* cannot be used as a multiplier, because it is a *quantity* and *not* a *number*.

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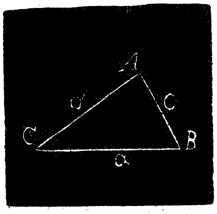
OBLIQUE ANGLED TRIANGLES.

By **RALPH H. KUNSTADTER, Graduate Student, Yale University, New Haven, Connecticut.**

Solution of an oblique angled triangle, given the two sides and the angle included between them.

The formula found below, was not known to me being in use, and have ascertained its existence only after having derived it originally in a simple way from the fundamental theorem of trigonometry, namely; the proportion of the sines of the angles is direct with that of the sides opposite to them.

Given in an oblique angled triangle for instance, the sides a, b, and angle C, and we wish to find directly from the given quantities the angle B.



$\frac{a}{b}=\frac{\sin A}{\sin B}\ldots\ldots(1)$. Equation (1) has two unknown quantities, we must therefore get rid of A, in order to solve our example. $\frac{a}{b}=\frac{\sin[180^\circ-(B+C)]}{\sin B}=\frac{\sin(B+C)}{\sin B}\ldots(2)$

$\frac{a}{b}=\frac{\sin B\cos C+\cos B\sin C}{\sin B}\ldots\ldots(3)=\cos C+\cot B\sin C\ldots(4)$. $\cot B=\frac{a}{b\sin C}-\cot C\ldots\ldots(5)$. In order to make equation (5) convenient for the application of logarithmic tables, assume $\frac{a}{b\sin C}=\cot\phi$. Hence,